

# A NOTE ON THE GREAT MATHEMATICIAN OF RAMANUJAN

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**Abstract:** The great mathematician Srinivasa Ramanujan has said about the number theory, mathematical analysis, infinite series and continued fractions including solutions to mathematical problems then considered unsolvable. A galaxy to mathematician flourished in Indian mathematics subcontinent during the period from 1887 AD to 1920 AD, who left their indelible mark on the history of mathematical sciences.

These great scholar of mathematical sciences enriched the knowledge of the decimal number system, negative number, Arithmetic, Algebra, Astronomy, Geometry (Plane and spherical), Trigonometry and even differential calculus. The Indian mathematician made early contribution to the study of the concept of zero as a number theory. During his short life Srinivasa Ramanujan independently compiled nearly 3900 results (mostly identities and equations)<sup>1</sup>. Many were completely novel, his original and highly unconventional results, such as the Ramanujan prime, the Ramanujan theta function, partition formulae and Mock theta functions have opened entire new areas of work and inspired a vast amount of further research<sup>2</sup>.

In the present paper, we have made effort to high-light some salient features of the mathematical sciences and its various application made by the development of Indian mathematics for the benefit of our new generation.

**Keywords:** Pure Mathematics, Mathematical Analysis, Lost Notebook, Convergent Series, Arithmetic Progression, Heegner Number, Partition Functions, Infinite Series.

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## 1. INTRODUCTION

After the glorious names of Aryabhata and Bhaskara, there came on the horizon of Indian mathematics, the outstanding genius of Srinivasa Ramanujan born was 22 December 1887 in Erode, Madras Presidency, British India, whose works are being pursued with great zeal the world over during his birth centenary<sup>3</sup>. He was an Indian mathematician who lived during the British Rule in India. He contributed much to the development of mathematical sciences, specially in the field of pure mathematics. His substantial contribution in the field of mathematical analysis, number theory, infinite series and continued fractions were marvelous. Srinivasa Ramanujan who his monumental work : Ramanujan's sum, Ramanujan prime, Landau-Ramanujan constant, Mock theta functions, Ramanujan conjecture, Ramanujan - Soldner constant, Ramanujan theta function, Rogers- Ramanujan identities, Ramanujan's master theorem and Ramanujan-Sato series and the Ramanujan Journal a Scientific Journal was established to publish work in all areas of Mathematics influenced by Ramanujan<sup>4</sup> and his notebooks containing summaries of his published and unpublished results have been analysed and studied for decades since his death as a source of new mathematical ideas.

Srinivasa Ramanujan initially developed his own mathematical research in isolation : according to Hans Eysenck- "He tried to interest the leading professional mathematicians in his work but failed for the most part. What he had to show them was too novel, too unfamiliar and additionally presented in unusual ways, they could not be bothered"<sup>5</sup>. Seeking mathematician who could better understand his work in 1913 AD he began a postal correspondence with the English mathematician G.H. Hardy at the University of Cambridge, England.

It is said that G. H. Hardy was astonished at the originality and profundity of the very first results communicated to him by Srinivasa Ramanujan. Such extraordinary skill and inventiveness in the manipulation of complicated formulas has occurred only a few times in the history of Mathematics: Euler, Gauss, Jacobi had such prowess. Ramanujan had no formal education in Mathematics. He left his proofs lacking rigour. But pioneers and path-finders, exploring boldly new terrians of mathematical thought, permit themselves a freedom in their attack on a mathematical problem. Their intuition often provides them with an innate feeling for what is correct and what is not. It is not a difficult task to edit the works of great savants of by gone ages so as to satisfy the current demands in mathematical exposition<sup>6</sup>.

Srinivasa Ramanujan’s work as extraordinary, G.H. Hardy arranged for him to travel to Cambridge. In his notes, Hardy commented that Ramanujan had produced ground breaking new theorems, including some that “defeated me completely, I had never seen anything in the least like them before”<sup>7</sup> and some recently proven but highly advanced results.

He became one of the youngest Fellows of the Royal Society and only the second Indian member and the first Indian to be elected a Fellow of Trinity College Cambridge of his original letters, G.H. Hardy stated that a single look was enough to show they could have been written only by a mathematician of the highest calibre, comparing Ramanujan to mathematical geniuses such as Euler and Jocabi. His last letters to Hardy, written in January 1920, show that he was still continuing to produce new mathematical ideas and theorems. His “lost notebook” containing discoveries from the last year of his life, caused great excitement among mathematicians when it was rediscovered in 1976AD.

Ramanujan was plagued by health problems throughout his life. His health worsened in England, possibly he was also less resilient due to the difficulty of keeping to the strict dietary requirements of his religion there and because of wartime rationing in 1914-18. He was diagnosed with tuberculosis and a severe vitamin deficiency and confined to a sanatorium. In 1919 he returned to Kumbakonam, Madras Presidency, British India and where he died in 26 April 1920 at the age of 32 years.

## 2. METHODOLOGY

Srinivasa Ramanujan rapidly convergent series for  $\pi$  have recently evoked great interest. The have been studied from diverse points of view. Ramanujan discovered several rapidly converged series for :

1.  $\frac{4}{\pi} = 1 + \frac{7}{4} \left(\frac{1}{2}\right)^3 + \frac{13}{4^2} \left(\frac{1 \times 3}{2 \times 4}\right)^3 + \frac{19}{4^3} \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^3 + \dots$
2.  $\frac{16}{\pi} = 5 + \frac{47}{64} \left(\frac{1}{2}\right)^3 + \frac{89}{64^2} \left(\frac{1 \times 3}{2 \times 4}\right)^3 + \frac{131}{64^3} \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^3 + \dots$

Herein {1,7,13,19,...} in (1) and {5, 47, 89, 131, .....} in (2) are all in arithmetic progression.

In Mathematics there is a distinction between insight and formulating or working through a proof. Ramanujan proposed an abundance of formulae that could be investigated later in depth. G.H. Hardy said that Ramanujan’s discoveries are unusually rich and that there is often more to them than initially meets the eye. As a by product of his work, new directions of research were opened up. Examples of the most intriguing of these formulae include infinite series for  $\pi$ , one of which is given below:

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

This result is based on the negative fundamental discriminant  $d = -4 \times 58 = -232$  with class number  $h(d) = 2$ . Further,

$$26390 = 5 \times 7 \times 13 \times 58 \text{ and } 16 \times 9801 = 396^2,$$

which is related to the fact that

$$e^{\pi\sqrt{58}} = 396^4 - 104.000000177 \dots$$

This might be compared to Heegner numbers, which have class number 1 and yield similar formulae.

Ramanujan series for  $\pi$  converges extraordinarily rapidly and forms the basis of some of the fastest algorithms currently used to calculate  $\pi$ . Truncating the sum to the first term also gives the approximation  $\frac{9801\sqrt{2}}{4412}$  for  $\pi$ , which is correct to six decimal places, truncating it to the first two terms gives a value correct to 14 decimal places. See also the more general Ramanujan-Sato series.

Srinivasa Ramanujan described an iterative procedure to determine the smallest root of equation<sup>8</sup> :

The smallest root of  $f(x) = 0$ , we consider  $f(x)$  in the form:

$$f(x) = 1 - (a_1 x + a_2 x^2 + a_3 x^3 + \dots),$$

and then write

$$[1 - (a_1 x + a_2 x^2 + a_3 x^3 + \dots)]^{-1} = b_1 + b_2 x + b_3 x^2 + \dots \quad (1)$$

$$\Rightarrow 1 + (a_1 x + a_2 x^2 + a_3 x^3 + \dots) + (a_1 x + a_2 x^2 + a_3 x^3 + \dots)^2 + \dots$$

$$= b_1 + b_2 x + b_3 x^2 + \dots \quad (2)$$

Comparing the coefficients of like powers of  $x$  on both sides of equation(2), we get

$$b_1 = 1$$

$$b_2 = a_1 = a_1 b_1, \text{ since } b_1 = 1$$

$$b_3 = a_2 + a_1^2 = a_2 b_1 + a_1 b_2, \text{ since } b_2 = a_1$$

$$\begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$$

$$\begin{aligned} b_k &= a_1 b_{k-1} + a_2 b_k + \dots + a_{k-1} b_1 \\ &= a_{k-1} b_1 + a_{k-2} b_2 + \dots + a_1 b_{k-1} \end{aligned}$$

The ratios  $\frac{b_1}{b_2}$ , called the convergents, approach, in the limit, the smallest root of  $f(x) = 0$

In 1918 G.H. Hardy and Srinivasa Ramanujan studied the partition function  $P(n)$  extensively. They gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer. In 1937 Hans Rademacher refined their formula to find an exact convergent series solution to this problem. Ramanujan and Hardy's work in this area gave rise to a powerful new method for finding asymptotic formulae called the circle method.

### 3. CONCLUSION

Here we see that it is a well established fact that knowledge of mathematical sciences including the concept of number theory, decimal number system, negative number, algebra, mathematical analysis, infinite series and continued fractions were marvelous. Srinivasa Ramanujan was one of India's greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions convergent series for  $\pi$ , infinite series and theta function. Ramanujan continued to develop his mathematical ideas and began to pose problems and solve problems in the Journal of the Indian Mathematical Society. Ramanujan had no formal education in mathematics.

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